Learning Objectives:
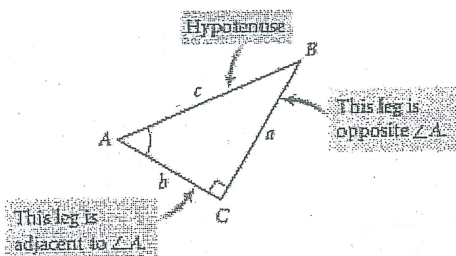
1. Define the trigonometric ratios sine, cosine, and tangent.
2. Use trigonometric ratios to find the unknown lengths of the sides of a right triangle.
3. Use inverse trigonometric ratios to find the unknown angle measures of a right triangle.

Trigonometric Ratios

For any acute angle A in a right triangle, the sine of $\angle A$ is the ratio of the length of the leg opposite $\angle A$ to the length of the hypotenuse.

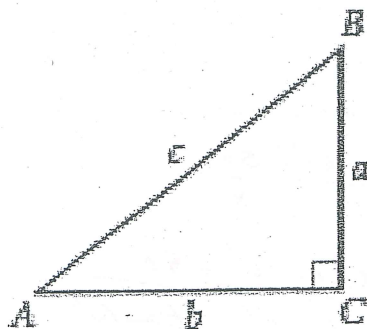$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$$

The cosine of $\angle A$ is the ratio of the length of the leg adjacent to $\angle A$ to the length of the hypotenuse.

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$$

The tangent of $\angle A$ is the ratio of the length of the opposite leg to the length of the adjacent leg.

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{a}{b}$$

Given the triangle below, complete the following table:

$$\sin A = \frac{o}{h} = \frac{a}{c}$$

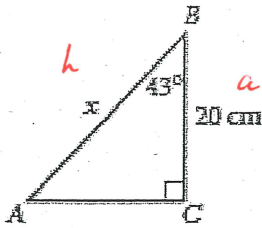$$\cos A = \frac{a}{h} = \frac{b}{c}$$

$$\tan A = \frac{o}{a} = \frac{a}{b}$$

$$\sin B = \frac{o}{h} = \frac{b}{c}$$

$$\cos B = \frac{a}{h} = \frac{a}{c}$$

$$\tan B = \frac{o}{a} = \frac{a}{b}$$
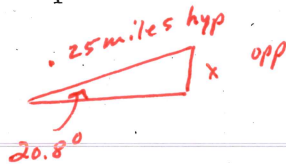
Example 1: Find the unknown length, x.

$$\frac{\cos 43°}{1} = \frac{20}{x}$$

$$\frac{x(\cos 43°)}{\cos 43°} = \frac{20}{\cos 43°}$$

$$\boxed{x \doteq 27.347}$$

Example 2: The steepest paved road in the world is Baldwin Street in Dunedin, New Zealand. It runs uphill for a quarter mile at an angle of 20.8° with the valley floor. How much higher are you at the top of the hill than at the bottom?
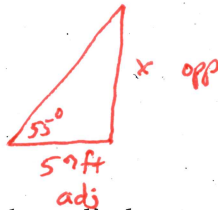

$$\sin 20.8° = \frac{x}{.25}$$

$$.25 \sin 20.8 = x$$

$$\boxed{.089 \text{ miles} = x}$$

Example 3: At 57 feet from the base of a building, you need to look up at a 55° to see the top of a building. What is the height of the building?
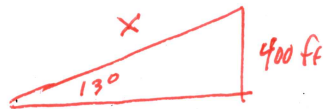

$$\tan 55° = \frac{x}{57}$$

$$57 \tan 55° = x$$

$$\boxed{81.404 \text{ ft} = x}$$

Example 4: An airplane climbs at an angle of 13° with the ground. What is the distance it has traveled when it has attained an altitude of 400 feet?
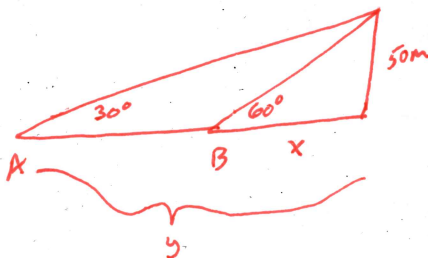

$$\sin 13° = \frac{400}{x}$$

$$\frac{x \sin 13°}{\sin 13°} = \frac{400}{\sin 13°}$$

$$\boxed{x = 1778.165 \text{ ft}}$$

Example 5: Adam and Brian are standing some distance apart on the same side of a building 50 meters tall. From where Adam stands, the angle of elevation of the top of the building is 30°. From where Brian stands, the angle is 60°. What is the distance between Brian and Adam?

$$\frac{\tan 30°}{1} = \frac{50}{y} \qquad \frac{\tan 60°}{1} = \frac{50}{x}$$

$$\frac{y \tan 30°}{\tan 30°} = \frac{50}{\tan 30°} \qquad \frac{x \tan 60°}{\tan 60°} = \frac{50}{\tan 60°}$$

$$y = 86.603 \qquad x = 28.868$$

$$\boxed{y - x = 57.735 \text{ m}}$$

Example 6: Michael, whose eyes are six feet off the ground, is standing 36 feet away from the base of a building, and he looks up at a 50° angle of elevation to a point on the edge of the buildings roof. How tall is the building?

$$\tan 50° = \frac{x}{36}$$

$$\boxed{x + 6 = 48.903 \text{ ft}}$$

$$36 \tan 50° = x$$

$$42.903 = x$$
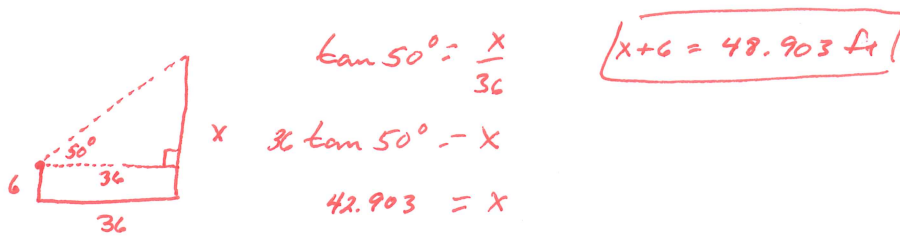
Example 7: When the plane had flown 4,150 feet from the airport where it had taken off, it had covered a horizontal distance of 3,660 feet. What is the angle at which the plane rose from the ground?

4,150 ft

3,660 ft

$$\cos\theta = \frac{3660}{4150}$$

$$\theta = \cos^{-1}\left(\frac{3660}{4150}\right)$$

$$\boxed{\theta = 28.124°}$$
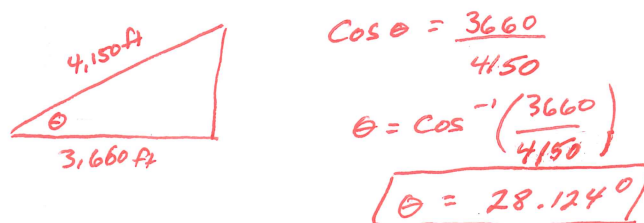
Example 8: Two hikers leave their campsite. Emily walks east 2.85 km and Savannah walks South 6.03 km. After Savannah gets to her destination, she looks directly toward Emily's destination. What is the measure of the angle between the path Savannah walked and her line of sight to Emily's destination?

2.85 km

6.03 km

$$\tan\theta = \frac{6.03}{2.85}$$

$$\theta = \tan^{-1}\left(\frac{2.85}{6.03}\right)$$

$$\boxed{\theta = 25.697°}$$
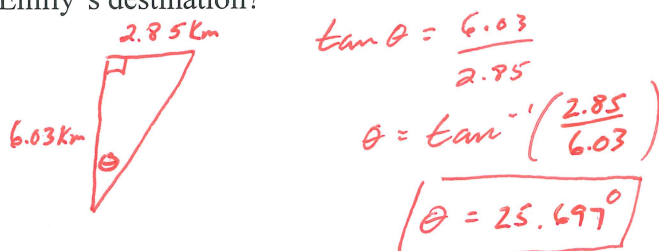
Example 9: Johnny has a 6-foot piece of plywood to use to make a bicycle ramp. If he places the end of the ramp on a pile of bricks that is 2 feet high, what angle does the other end of the ramp make with the ground?

6 ft   2 ft

$$\sin\theta = \frac{2}{6}$$

$$\theta = \sin^{-1}\left(\frac{1}{3}\right)$$

$$\boxed{\theta = 19.471°}$$
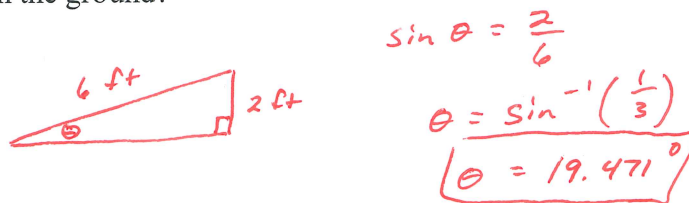
Example 10: A radio station antenna is supported by wire cables, which are attached to the top of the antenna. The points where the cables are anchored to the ground are 250 feet away from the antenna. If the cables are 610 feet long, find the measure of the angles formed by the antenna and the cables.

610 ft

250 ft

$$\sin\theta = \frac{250}{610}$$

$$\theta = \sin^{-1}\left(\frac{25}{61}\right)$$

$$\boxed{\theta = 24.195°}$$